Clasa a VI-a- Bareme

Problema 1

Găsiți cifrele nenule a, b, c, astfel încât

$$22{,}5 \cdot [\,\overline{a,b(c)} - \overline{a,c} + \overline{b,c(a)} - \overline{b,a} + \overline{c,a(b)} - \overline{c,b}\,] = 1.$$

Nicolae Ivăşchescu, Craiova, Gazeta Matematică 2/2014 – Supliment

Barem:

$\overline{a,b(c)} = a + \frac{\overline{bc}-b}{90}$

$\overline{a,c} = \frac{\overline{ac}}{10}$ 1p

Arată că:

$\overline{a,b(c)} - \overline{a,c} = a + \frac{\overline{bc}-b}{90} - \frac{\overline{ac}}{10}$

$= \frac{90a}{90} + \frac{10b+c-b}{90} - \frac{90a+9c}{90}$ 1p

$= \frac{90a+10b+c-b-90a-9c}{90} = \frac{9b-8c}{90}$ 1p

$\overline{b,c(a)} - \overline{b,a} = \frac{9c-8a}{90}$ 1p

$\overline{c,a(b)} - \overline{c,b} = \frac{9a-8b}{90}$ 1p

Ecuația devine: $\frac{225}{10} \cdot \left(\frac{9b-8c}{90} + \frac{9c-8a}{90} + \frac{9a-8b}{90}\right) = 1$

$\Leftrightarrow \frac{225}{10} \cdot \frac{a+b+c}{90} = 1$

$\Leftrightarrow a+b+c = 4$ 1p

Din $\left.\begin{matrix} a+b+c=4 \\ a,b,c \neq 0 \end{matrix}\right\} \Rightarrow (a,b,c) \in \{(1,1,2);\ (1,2,1);\ (2,1,1)\}$ 1p